A Two Phase Investment Game for Competitive Opinion Dynamics in Social Networks*

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ABSTRACT

We study the problem of two competing camps aiming to maximize the adoption of their respective opinions, by optimally investing in nodes of a social network in two phases. The final opinion of a node in phase 1 acts as its bias in phase 2, and this bias determines the effectiveness of a camp's investment on the node. Using an extension of Friedkin-Johnsen model of opinion dynamics, we formulate the camps' utility functions. We show the existence and polynomial time computability of Nash equilibrium under reasonable assumptions. Using simulations, we quantify the effects of the nodes' biases and the weightage attributed to them, as well as that of a camp deviating from its equilibrium strategy.

1 INTRODUCTION

We consider two competing camps with positive and negative opinion values (referred to as good and bad camps respectively), aiming to maximize the adoption of their respective opinions in a social network. With the opinion adoption quantified as the sum of opinion values of all nodes [19, 20], the good camp aims to maximize this sum while the bad camp aims to minimize it. Since nodes update their opinions based on their neighbors' opinions [1, 14], a camp would want to influence the opinions of influential nodes by investing on them. Thus given a budget constraint, the strategy of a camp comprises of: how much to invest and on which nodes, in presence of a competing camp which would also invest strategically.

Motivation. In Friedkin-Johnsen model [15, 16], a node's bias plays a critical role in determining its final opinion, and consequently the opinions of its neighbors, and so on. If nodes give significant weightage to their biases, the camps would want to influence these biases. This could be achieved by campaigning in two phases, wherein a node's opinion at the end of phase 1 would act as its biased opinion in phase 2. Further, a node's bias often impacts a camp's effectiveness on that node. If a node's bias is positive, the good camp's investment on it is likely to be more effective than the bad camp's. The reasoning is on similar lines as the bounded confidence model [25], wherein a node pays more attention to opinions that do not differ too much from its own. Hence, a camp's investment in phase 1 not only gives it a head start in phase 2 by influencing the biases, but also enhances the effectiveness of its investment in phase 2. With the possibility of campaigning in two phases, a camp could not only decide which nodes to invest on, but also how to split its available budget between the two phases.

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Related Work. Problems related to maximizing opinion adoption in social networks have been extensively studied [11, 14, 18, 21, 24, 26, 28]. The competitive setting has resulted in several game theoretic studies [2, 4, 8, 17]. For analytically tractable models such as Friedkin-Johnsen, there have been studies to determine optimal investments on nodes [5, 8, 13, 20]. Our work extends these studies to two phases, by identifying influential nodes in the two phases and how much they should be invested on in each phase.

There have been a few studies on adaptive selection of influential nodes in phases [3, 6, 7, 12, 22, 29–33]. While the reasoning behind phases in these studies is to adaptively select nodes based on previous observations, we use them for influencing nodes' biases; this necessitates a very different conceptual and analytical treatment. In our earlier work [9], we assumed the effectiveness of a camp's investment on a node to be independent of the node's bias. This work relaxes that assumption and undertakes a more general study.

2 OUR MODEL

We represent social network as a weighted directed graph, with set of nodes V. Table 1 presents the notation. As our opinion dynamics runs in two phases, most parameters have two values, one for each phase. For such a parameter, we denote its value corresponding to phase p using superscript (p). In our setting, the bias of node i in phase 2 is $z_i^{(1)}$, which is its opinion value at the end of phase 1.

Since a node's bias impacts the effectiveness of camps' investments on it, $z_i^{(p-1)} > 0$ would likely lead to $w_{ig}^{(p)} > w_{ib}^{(p)}$. As w_{ii}^0 quantifies the weightage given by node i to its bias, we propose the following natural model (where $w_{ig}^{(p)} + w_{ib}^{(p)} = \theta_i$): $w_{ig}^{(p)} = \frac{\theta_i}{2}\left(1 + w_{ii}^0 z_i^{(p-1)}\right)$ and $w_{ib}^{(p)} = \frac{\theta_i}{2}\left(1 - w_{ii}^0 z_i^{(p-1)}\right)$.

Table 1: Notation

z_i^0	initial biased opinion of node i in phase 1
w_{ii}^0	weightage attributed by node i to its bias
w_{ij}	weightage attributed by node i to the opinion of node j
θ_i	total weightage attributed by node i to the camps' opinions
$w_{ig}^{(p)}$	weightage attributed by node i to good camp in phase p
$w_{ib}^{(p)}$	weightage attributed by node i to bad camp in phase p
$x_i^{(p)}$	investment made by good camp on node i in phase p
$y_i^{(p)}$	investment made by bad camp on node i in phase p
k_g	total budget of the good camp
k_b	total budget of the bad camp
$z_i^{(p)}$	opinion of node i at the end of phase p

As the influence of good camp on node i in phase p would be an increasing function of its investment $x_i^{(p)}$ and weightage $w_{ig}^{(p)}$, we consider the influence to be $+w_{ig}^{(p)} x_i^{(p)}$. Similarly, $-w_{ib}^{(p)} y_i^{(p)}$ is the influence of bad camp (negative opinion) on node i. Considering budget constraints, the camps should invest in the two phases such that $\sum_{i \in V} \left(x_i^{(1)} + x_i^{(2)}\right) \leq k_g$ and $\sum_{i \in V} \left(y_i^{(1)} + y_i^{(2)}\right) \leq k_b$.

Generalizing Friedkin-Johnsen update rule to two phases and accounting for camps' investments, the update rule in phase p is:

$$\forall i \in V: \; z_i^{(p)} \leftarrow w_{ii}^0 z_i^{(p-1)} + \sum_{j \in V} w_{ij} z_j^{(p)} + w_{ig}^{(p)} x_i^{(p)} - w_{ib}^{(p)} y_i^{(p)}$$

Let $\Delta = (\mathbf{I} - \mathbf{W})^{-1}$, where $\mathbf{W}$ is the matrix consisting of weights w_{ij}. Let $b_{ji} = r_j w_{jj}^0 \Delta_{ji}$ and $c_i = w_{ii}^0 z_i^0$. Let $r_i = \sum_{j \in V} \Delta_{ji}$ and $s_i = \sum_{j \in V} r_j w_{jj}^0 \Delta_{ji}$. The sum of opinion values of the nodes at the end of phase 2 can be derived to be:

$$\begin{aligned}\sum_{i \in V} z_i^{(2)} = &\sum_{i \in V} \sum_{j \in V} c_i b_{ji} + \sum_{j \in V} x_j^{(2)} \frac{\theta_j}{2} \Big(\sum_{i \in V} c_i b_{ji} + r_j \Big) + \sum_{j \in V} y_j^{(2)} \frac{\theta_j}{2} \Big(\sum_{i \in V} c_i b_{ji} - r_j \Big) \\ &+ \sum_{i \in V} x_i^{(1)} \frac{\theta_i}{2} (1 + c_i) \Big(s_i + \sum_{j \in V} x_j^{(2)} \frac{\theta_j}{2} b_{ji} + \sum_{j \in V} y_j^{(2)} \frac{\theta_j}{2} b_{ji} \Big) \\ &- \sum_{i \in V} y_i^{(1)} \frac{\theta_i}{2} (1 - c_i) \Big(s_i + \sum_{j \in V} x_j^{(2)} \frac{\theta_j}{2} b_{ji} + \sum_{j \in V} y_j^{(2)} \frac{\theta_j}{2} b_{ji} \Big) \quad (1)\end{aligned}$$

Two-phase Katz Centrality. r_i resembles Katz centrality of node i [23], capturing its influencing power over other nodes in a single phase setting (corresponds to phase 2 in two-phase setting since it is the terminal phase). However, while selecting nodes to invest on in phase 1, the effective power of node i depends on its influencing power over those nodes (j), which would give good weightage (w_{jj}^0) to their biases in phase 2, as well as have a good influencing power over other nodes (r_j) in phase 2. This is captured by s_i, and can be interpreted as the two-phase Katz centrality.

3 PROBLEM FORMULATION AND RESULTS

Let $\mathbf{x}^{(\mathbf{p})}$ and $\mathbf{y}^{(\mathbf{p})}$ be the vectors consisting of $x_i^{(p)}$ and $y_i^{(p)}$, respectively. Hence, $(\mathbf{x}^{(1)}, \mathbf{x}^{(2)})$ and $(\mathbf{y}^{(1)}, \mathbf{y}^{(2)})$ are the strategies of the good and bad camps. Let $u_g(\cdot)$ and $u_b(\cdot)$ be their respective utilities. The good camp aims to maximize the value of Equation (1), while the bad camp aims to minimize it. Hence, the problem is:

Find Nash equilibrium, given that

$$u_g\big((\mathbf{x}^{(1)}, \mathbf{x}^{(2)}), (\mathbf{y}^{(1)}, \mathbf{y}^{(2)})\big) = \sum_{i \in V} z_i^{(2)} \;,\; u_b\big((\mathbf{x}^{(1)}, \mathbf{x}^{(2)}), (\mathbf{y}^{(1)}, \mathbf{y}^{(2)})\big) = -\sum_{i \in V} z_i^{(2)}$$

subject to

$$\sum_{i \in V} \left(x_i^{(1)} + x_i^{(2)}\right) \leq k_g \;,\; \sum_{i \in V} \left(y_i^{(1)} + y_i^{(2)}\right) \leq k_b \;, \forall i \in V: x_i^{(1)}, x_i^{(2)}, y_i^{(1)}, y_i^{(2)} \geq 0$$

Optimal Strategy in Absence of Competing Camp. In absence of the bad camp ($y_i^{(1)} = y_i^{(2)} = 0, \forall i \in V$ in Equation (1)), let the good camp split its budget k_g such that $k_g^{(1)}$ and $k_g^{(2)}$ are the respective investments in phases 1 and 2. It can be shown that, in the search space $k_g^{(1)} + k_g^{(2)} \in (0, k_g]$, it is optimal for the good camp to exhaust its entire budget ($k_g^{(1)} + k_g^{(2)} = k_g$), and to invest on at most one node in each phase. Consider α and β to be the candidate nodes to invest on in phases 1 and 2 respectively. It can be derived that the optimal $k_g^{(1)} \in (0, k_g]$ for node pair (α, β) is:

$$\min \left\{ \max \left\{ \frac{k_g}{2} + \frac{s_\alpha}{\theta_\beta r_\beta w_{\beta\beta}^0 \Delta_{\beta\alpha}} - \frac{1 + w_{\beta\beta}^0 \sum_{i \in V} \Delta_{\beta i} w_{ii}^0 z_i^0}{\theta_\alpha w_{\beta\beta}^0 \Delta_{\beta\alpha} (1 + w_{\alpha\alpha}^0 z_\alpha^0)}, 0 \right\}, k_g \right\}$$

and the corresponding optimal $k_g^{(2)} \in (0, k_g]$ is $k_g - k_g^{(1)}$. Intuitively, a high s_α encourages a high investment in phase 1, while a high r_β drives the investment towards phase 2. Among the $|V|^2 + 1$ possible node pairs, namely, $(\alpha, \beta) \in V \times V \cup \{(\phi, \phi)\}$ where (ϕ, ϕ) captures $k_g^{(1)} = k_g^{(2)} = 0$, we determine the pair (α^*, β^*) that maximizes Equation (1). Thus, the resulting optimal strategy is to invest the optimal $k_g^{(1)}$ (corresponding to pair (α^*, β^*)) on node α^* in phase 1, and the corresponding optimal $k_g^{(2)}$ on node β^* in phase 2.

Nash Equilibrium Strategy Profile. Similar to above, we first show that it is optimal for either camp to invest on at most one node in each phase. Consider (α, β) and (γ, δ) to be the candidate node pairs for the good and bad camps, respectively. For a given node profile $((\alpha, \beta), (\gamma, \delta))$, we derive $k_g^{(1)}$ and $k_b^{(1)}$ by determining the saddle point of Equation (1). For this, we make practically reasonable assumptions: $w_{ij} \geq 0, \forall (i, j)$ and $w_{ii}^0 \geq 0, \theta_i \geq 0, z_i^0 \in [-1, 1], \forall i \in V$, under which $\sum_{i \in V} z_i^{(2)}$ is concave w.r.t. $k_g^{(1)}$ and convex w.r.t. $k_b^{(1)}$. On deriving $k_g^{(1)}, k_g^{(2)}, k_b^{(1)}, k_b^{(2)}$ corresponding to the saddle point for node profile $((\alpha, \beta), (\gamma, \delta))$, we compute the value of Equation (1). Consequently, we create a transformed two-player zero-sum game with each player having $|V|^2 + 1$ pure strategies, where each pure strategy corresponds to a node pair. In such a game, Nash equilibrium exists and can be found efficiently using linear programming [27]. The elaborate methodology including the expressions for $k_g^{(1)}, k_g^{(2)}, k_b^{(1)}, k_b^{(2)}$ are provided in [10].

Simulation Results. Following are some highlights of the simulation results observed on real-world network datasets:

- A low investment in phase 1 results in poor biases for phase 2, resulting in unfavorable opinion values to start with in phase 2, and also less effectiveness of investment in phase 2. A high investment in phase 1 spares less budget for phase 2, resulting in the inability to fully harness the influenced biases. Hence, there is a tradeoff.
- Higher w_{ii}^0's drive the camps to invest more in phase 1 so as to have favorable opinions at the end of phase 1 and hence favorable biases for phase 2. This plays a key role in determining the final opinion owing to the high weightage attributed to biases in phase 2 and also the enhanced effectiveness of investments in phase 2.
- Higher z_i^0's result in a better utility for the good camp (and worse utility for the bad camp), not only because of the head start, but also due to the good camp's investments being more effective.
- The loss incurred by deviating from Nash equilibrium strategy to a myopic strategy (investing the entire budget in phase 1) is significant for low w_{ii}^0's, since in this range, it is actually optimal to invest most of the budget in phase 2. For high w_{ii}^0's, however, we observe that the equilibrium strategy is to invest heavily in phase 1, thus resulting in myopic strategy not being a poor choice. On the other hand, if a camp deviates to a strategy, which is its optimal one in absence of the competing camp, the loss is observed to be relatively less for the entire range of w_{ii}^0's. To arrive at more generalizable results, further efficient algorithms of computing Nash equilibrium are warranted, for simulations with larger networks.

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